Computing self-similar solutions to the 1D Nonlinear Schrödinger Equation with a power-law nonlinearity

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The BVP we want to solve is:

$$\frac{d^2Q_r}{dx^2} + \left(Q_r^2 + Q_i^2\right)^{\sigma} Q_r - Q_r - G\left(\frac{Q_i}{\sigma} + x\frac{dQ_i}{dx}\right) = 0, \tag{1a}$$

$$\frac{d^2Q_i}{dx^2} + \left(Q_r^2 + Q_i^2\right)^{\sigma} Q_i - Q_i + G\left(\frac{Q_r}{\sigma} + x\frac{dQ_r}{dx}\right) = 0, \tag{1b}$$

supplemented with zero Neumann BCs at $x = \pm L$. The parameter σ is set to a specific value but the parameter G is determined by the orthogonality condition:

$$\langle u - T(x), T(x)/\sigma + x\frac{dT}{dx}\rangle = 0, \tag{2}$$

where $T(x) = (1+\delta)^{\frac{1}{2\delta}} \operatorname{sech}^{\frac{1}{\delta}}(\delta x)$ with $\delta = 2$. The system given by Eqs. (1a)-(1b) and (2) is closed now. The initial guess for Q_r is $Q_r^{(0)} = (1+\sigma)^{\frac{1}{2\sigma}} \operatorname{sech}^{\frac{1}{\sigma}}(\sigma x)$, and $Q_i^{(0)} = 0$.

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