Weak Formulation

monirulislam4566

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1 Variational formulation(using Backward-Euler)

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\begin{aligned}
(u^{n+1}-u^n,\phi) \quad &+ \quad dt(\nabla w^{n+1},\nabla\phi)-(g,\phi)+dt\sum_{e\in\varepsilon^I}\int_e\Bigg[[[w^{n+1}]]\{\{\frac{\partial\phi}{\partial n}\}\}+4.5.\frac{[[w^{n+1}]]}{h_e}[[\phi]]+[[\phi]]\{\{\frac{\partial w^{n+1}}{\partial n}\}\} \\
&+ \quad \frac{1}{40}h_e[[\frac{\partial^2 w^{n+1}}{\partial n^2}]]\ [[\phi]]\Bigg]ds-dt\sum_{e\in\varepsilon^D}\int_e w^{n+1}\frac{\partial\phi}{\partial n}\ ds+(w^{n+1},\psi)-0.01(\nabla u^{n+1},\nabla\psi) \\
&+ \quad \sum_{e\in\varepsilon^I}\int_e\Bigg[[[u^{n+1}]]\{\{\frac{\partial\psi}{\partial n}\}\}+4.5.\frac{[[u^{n+1}]]}{h_e}[[\psi]]+[[\psi]]\{\{\frac{\partial u^{n+1}}{\partial n}\}\} \\
&+ \quad \frac{1}{40}h_e[[\frac{\partial^2 u^{n+1}}{\partial n^2}]]\ [[\psi]]\Bigg]ds-\sum_{e\in\varepsilon^D}\int_e u^{n+1}\frac{\partial\psi}{\partial n}\ ds-(f(u^{n+1}),\psi)=0
\end{aligned}
$$

Here, ε^I is the collection of all interior edges and ε^D is the collection of all boundary edges and h_e is the charcteristic length of the edge e (which we can incorporate using lenEdge in FreeFem++). Here, $f(u)=u^3-u$ is the non-linear term. We have used newton's method to handel non-linearity. This equation i used to form the bilinear form but adding internal edges on weak form giving error on my programming. How, i incorporate the internal edges please tell??. Also, i have to find the L^∞ error. Please share how to find L^∞ error for the solution u using my codes. Also, please help how to write gnuplot in my codes. Please help. It is urgent.

Here, $[[w]]$=jump of w and $\{\{\phi\}\}$ is the average of ϕ and simillarly for others symbols, n is the outward normal.